Inferring regional aquifer parameters by spectral analysis of groundwater head fluctuations A feasibility study



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MOTIVATION AND METHODOLOGY

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Key Points

Spectral analysis - regional aquifer parameters - stochastic analysis - deterministic analysis - field study - complex numerical analysis

Climate change is projected to have severe impacts on the amount and dynamics of groundwater recharge and consequently on water resources stored in aquifers. Projecting the response of regional aquifer systems to changing climate conditions using regional groundwater models is paramount. However, parameterizing the large-scale GW models are challenging due to scarcity of observational records as well as mismatch between scales of modeling and measurements.

In this work, we propose to bridge the scale gap and derive regional scale hydraulic parameters by spectral analysis of groundwater head fluctuations.

Liang et al. (2013) derived a semi-analytical solution of power spectral density of heads (PSD) for an unconfined aquifer, bounded by a no-flow boundary condition at x = 0 and a constant head boundary condition at x = L. Normally, the PSD determines the frequency response of an endless signal but observations of groundwater levels are limited in time and have a certain sampling frequency. Taking limited time series with too large intervals between each sample could lead to estimates of the PSD where certain frequencies might have been lost. Furthermore, the domain was assumed to be homogeneous, i.e. the hydraulic aquifer properties (specific yield Sy) and transmissivity T) were kept constant in the entire domain. In order to understand the potential and limitation of the developed semi-analytical solution, especially when applying it to the complex heterogeneous nature of aquifer systems, we first tried to gain experience in a numerical environment where input parameters are precisely known.

This synthetic study was conducted with a numerical model (https://www.opengeosys.org/) solving the 3D groundwater equation. The domain was reduced in dimensions to a vertical 2D transect so that finally two dimensions of possible flow orientation remained, whereas the semi-analytical solution was developed based on a 1D, horizontal flow problem. Allowing for vertical flow components in the numerical model represents the first level of complexification in comparison to the semi-analytical solution. This scenario was realized with a homogeneous model setup (section 2D Numerical Transects). We continued the synthetic study with log-normally distributed stochastic hydraulic conductivity fields. In a final deterministic experiment, the aquifer was divided into two different sections, each with a distinct hydraulic conductivity.

We applied a transient recharge signal to the numerical models and simulated the flow over a period of 30 years. The groundwater response signals at different locations in the aquifer and as an arithmetic mean over the thickness were transformed from the time domain to the frequency domain with the Fourier transform and the power spectral density was calculated (see Theory). The PSD was fitted with the semi-analytical solution of the head spectrum from Liang et al. (2013). The solution depends on the transmissivity T, specific yield S_y, aquifer length L and position x of the groundwater observation well along the transect. The aquifer length and the position of the measurement location were taken from the model setup so that the the fitting procedure aimed at optimizing the aquifer parameters T and S_y. The derived output parameters were compared with the model input parameters so that the validity of the analytical solution in complex synthetic environments could be estimated.

In addition to this synthetic study, we applied the spectral analysis to four groundwater observation wells in the Main catchment (section Field Study), derived the aquifer parameters and evaluated the sensitivity of the spectral analyses.

In a last experiment, we performed the spectral analysis on groundwater head time series modelled by a distributed 3D numerical model (section 3D Numerical Model). We were able to link the model's parameterization with the derived hydraulic conductivity by spectral analysis.

2D NUMERICAL TRANSECTS

Numerical Setup

The numerical experiments were conducted in synthetic 2D aquifers (transects) with different complexity, i.e. different distribution of aquifer properties and changing forcing. The models were set up with OpenGeoSys (https://www.opengeosys.org/) (Kolditz 2012) and ogs5py

(https://zenodo.org/record/3738563#.X7-9_qpKhfU) (Müller 2020). Figure 2 (a) depicts the forcing, geometry and boundary conditions and represents the homogeneous scenario. We continued with log-normal distributed hydraulic conductivity fields (figure 2 (b)) and investigated the spectral analysis afterwards in a deterministic block mdel (figure 2 (c)).

We chose a white noise recharge for all setups (a, b, c) and extended the analysis with a temporal correlated recharge in the deterministic setup (c).



Figure 2: (a) Aquifer geometry, forcing and boundary conditions for all setups (b) One realization of each correlation length of an isotropic, log-normal distributed hydraulic con- ductivity field generated from a Gaussian covariance model with variance 1. Top: 15 m, bottom: 5 m correlation length. (c) Deterministic block aquifer with two zones of hydraulic conductivity. The position of the boundary between the zones was varied throughout the investigation.

Numerical Results

Homogeneous Domain

We sampled several hundred combinations of T and S to obtain synthetic aquifers with different characteristic times and found out that the performance of the spectral analysis just depends on t_c in relation to the length of the time series (modeling period), thus we plotted the error in the derived parameters S and T against different characteristic times:



Figure 3: Error of deviation between input (numerical model) and output (spectral analysis) aquifer parameters with respect to input parameters plotted against the characteristic time tc of the aquifer for five locations. (a) Error of storativity. (b) Error of transmissivity.

Stochastic Log-Normal Hydraulic Conductivity

We generated 200 realizations of the conductivity field with gstools (https://zenodo.org/record/3751743#.X7--PKpKhfU) (Müller and Schüler 2020) for each correlation length (5 m and 15 m), ran the models, extracted the head time series, performed the spectral analysis at different locations and calculated the kernel density of the derived aquifer parameters S and T for each ensemble:



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Figure 4: Kernel density estimate for determined aquifer properties by spectral analysis of groundwater heads for 200 realizations of a stochastic (log-normal distributed K-field) aquifer, evaluated at different locations. Red and blue indicate 5 m and 15 m correlation length, respectively. The characteristic time t_c considering the geometric mean 108 days. (a) Derived storativities. The input storativity of the numerical model was held constant throughout all realizations (S = 0.03). (b) Derived transmissivities. The arithmetic, geometric and harmonic mean of the underlying distribution are depicted with horizontal lines.

Deterministic block model

In the deterministic block model we shifted the boundary between two zones of different hydraulic conductivity step wise. Figure 5 the two extreme scenarios and an intermediate setting:



Figure 5: Input (numerical model) and output (spectral analysis) transmissivity for six different deterministic aquifers. A white noise and a realistic, temporally correlated recharge generated by mHM was assigned to the numerical model. a) A low conductive zone in the upstream part of the aquifer and a high conductive zone at the outlet b) A high conductive part upstream and a low conductive part downstream.

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FIELD STUDY

Geological Setting and Data Basis

We selected 4 groundwater observation wells in the catchment of the Main river in central Germany and performed the spectral analysis on these time series.

Birkach and *Strullendorf Nord* are screened in deeper Burg-und Blasensandstein whereas *Stegaurach* and *Strullendorf West* are screened in quaternary sediments.



Figure 6: Subordinate catchment in the center of the Main catchment in central germany. Topography is indicated with black line (no labels) and the geology with colored layers. Bottom, right: Zoom closer to the four investigated groundwater observation wells and the valley with the Regnitz river.



We took simulated recharge time series (mHM, Samaniego et al. 2010) for each observation well. The intensity and delayed reaction of the fluctuations of the head time series differ due to different hydrogeological settings.

Figure 7: Groundwater head time series of four observation wells in the central Main catchment and corresponding recharge modeled with mHM.

Spectral Analysis

The power spectrum of the recharge and the head time series were obtained and compared. The shape of the spectra from *Birkach* and *Strullendorf Nord* are suprisingly similar indicating similar hydrological regimes. Shapes of spectra from *Stegaurach* and *Strullendort West* significantly differ from the others.



Figure 8: Power spectra of recharge (a) and groundwater head time series (b) for each observation well. For a better view, geologically similar wells are plotted in one figure: (c) Screened in Burg- und Blasensandstein, (d) screened in quaternary sediments.

The aquifer length L and the position of the head measurement x (distance to water divide) are parameters of the semi-analytical solution of the head spectrum (Liang et al. 2013) as well as T and S. Since L and x are difficult to estimate precisely in field studies, we performed a sensitivity analysis of the fitting of the head spectrum. We sampled roughly 250 combinations of L and x and fitted the solution to the spectra for each combination. Furthermore, we added a maximum and minimum pair of L and x for each well which was estimated with geological expertise.



Figure 9: Scatter plots of derived storativity S, transmissivity T, and resulting characteristic time t_c for different combinations of aquifer length L and distance to water divide x. The goodness of the fit was evaluated with the R^2 value. Color bars of S and T have the same log-scale for every observation point, whereas the linear scale for t_c and R^2 was adjusted in each plot. The big stars represent the manually added samples.

The power spectra of the groundwater level fluctuations were fitted with the semi-analytical solution and depicted in figure X. More weight was given to the lower frequencies, since the long term behavior of the aquifer is in scope of interest.



Figure 10: Spectra of groundwater heads (S_{hh} , solid) and a corresponding fit of the analytical solution (S_{hh} fit, dots) with the best R^2 value, selected from the two manually added samples.

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3D NUMERICAL MODEL

Model Domain

The Schäfertal is a small (1.4 km^2) agricultural catchment in the Hartz Mountains in Germany with 13 observation wells and a gauge at the outlet. The white transects represents the groundwater flow direction towards each well. The length L and the distance of the well to the water divide x are parameters of the semi-analytical solution from Liang et al. (2013) and are required for spectral analysis.



Figure 11: Agricultural watershed of the Schäfertal catchment with observation wells and the corresponding transects along the groundwater flow direction. Areal image taken from GoogleEarth.

A fully coupled hydrogeological model for the Schäfertal was set up with Hydrogeosphere solving the Richard's equation in the subsurface. The model was constructed and calibrated by Yang (2018). It consists of two layers with 5 zones differing in hydraulic properties. The thickness of the layers is roughly 0.5 and 5 m for upper and lower layer, respectively.



Figure 12: Numerical model of the Schaefertal catchment with 2 layers. Each layers consists of 5 zones with different hydraulic properties. The covered area of zone 1 and 6, 2 and 7 etc. match up vertically (scaled in Z with factor 5).

The laterally constant recharge was extracted from the model taking the amount of water which enters the deeper layers, thus accumulating in highly saturated conditions. In order to obtain longer time series, the model was looped four times resulting in roughly 40 years of modeling period.



Figure 13: Groundwater level fluctuations for observation wells 30b, 34, 35, 36, 52 and 53.



Figure 14: Groundwater level fluctuations for observation wells 57, 57a, 58a, 61, 62, 66 and 74.

The difference between recharge and discharge is due to small percentage of overland and interflow.



Figure 15: Recharge and discharge time series from 1998 to 2007.

Spectral Analysis

The power spectra for all observation wells were calculated. Except for well 30b and 36, the shape of the spectra can be fitted very well with the analytical solution.



Figure 16: Power spectra of the observation wells (except for well 35).

Since we aimed at linking the derived aquifer parameter T by spectral analysis to the distributed model input parameters, we sliced the model along the determined transects, assuming that water which drains towards each well travels predominantly along the corresponding slice. In this work, we focused on the hydraulic conductivity.

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Figure 17: Slicing the numerical model. Each zone was parameterized with another hydraulic conductivity (see legend on the right).



Figure 18: Slices of the numerical model which correspond to the transects of the respective wells (scaled in Z with factor 10). Wells are depicted with a colored square (see legend on top right).



The vertical thickness of the layers were determined and taken to calculate the hydraulic conductivity from the obtained transmissivities of the spectral analysis.

Figure 19: A single slice wich corresponds to observation well 30b. The thickness b at the location of the observation well has been estimated to calculate the hydraulic conductivity K from the derived transmissivity T by the spectral analysis with K = T / b.

We calculated the percentage of each zone in the slices and colored the fraction according to the hydraulic conductivity.



Figure 20: Fraction of the respective zone for each slice. Top: Only the upper layers are considered. Center: Only the lower layers are considered. Bottom: Both, upper and lower layers are considered. K (hydraulic conductivity) is given in m/s.

The derived hydraulic conductivity obtained from spectral analysis matches up with the weighted arithmetic mean of the model input hydraulic conductivity (compare red and black lines). Well 57a, 62 and 74 seems to be noticeably overestimated.



Figure 21: Results of the spectral analysis using thicknesses of only upper (SA_up), lower (SA_low) and combined (SA_comb) layers for every well and weighted, averaged hydraulic conductivities for each slice.

The overestimation of hydraulic conductivity for well 57a, 62 and 74 might be due to the changing saturation along the slices. In the center of the catchment and in the deeper soils, the saturation is much higher. The spectral analysis might be affected by partially saturated flow conditions.



Figure 22: Modeling domain (transparent) and slices with saturation for a single day. Towards the center of the catchment the saturation increases.

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THEORY

The Power Spectrum of Groundwater Heads

The basis of our analysis of hydrological time series is the Fourier transform. A signal in the time domain, e.g. measured groundwater levels over time, was transformed to the frequency domain signal with the forward Fourier transform. The semi-analytical solution from Liang (2013) was developed to estimate the power spectral density (PSD, power spectrum, spectrum) of the head fluctuations S_{hh} . The power spectral density characterizes a random process in the frequency domain and is usually calculated over several realizations. It gives the distribution of power in the signal over frequency and can be obtained in multiple ways. According to the Weiner-Khintchine theorem (Weiss 1960) the PSD for continuous signals can be obtained from the Fourier transform of the auto-covariance function R_{hh} :

$$S_{hh} = \int_{-\infty}^{\infty} R_{hh}(\tau) \ e^{-2\pi i f \tau} d\tau$$

Where τ is the time lag.

For the derivation of the semi-analytical head spectrum Liang (2013) considered an unconfined aquifer between a watershed and a river which can be approximated with the linearized Boussinesq equation (Bear 1972) evoking the Dupuit assumptions:

$$Trac{\partial^2 h}{\partial x^2} + W = S_y rac{\partial h}{\partial t}$$

 $h(x,0) = f(x); \qquad rac{\partial h}{\partial x}|_{x=0} = 0; \qquad h(L,t) = h_0$

where T $[L^2T^{-1}]$ is the transmissivity of the homogeneous and isotropic aquifer, h [L] is the groundwater level above a horizontal bottom of the aquifer, W $[LT^{-1}]$ is a source term representing recharge or evapotranspiration, S_y [-] is the specific yield, f(x) is initial condition, L [L] is the distance from the left no-flow boundary to the right constant-head boundary, h₀ is the water level at the constant-head boundary, x [L] is the coordinate, and t [T] is time (Liang 2013). They argue that the river fluctuations are in most cases relatively small compared to the water level fluctuations in the aquifer so that the boundary effect is limited to a small zone close to the constant-head boundary condition. Thus, they set T, Sy, f(x) and h₀ to be constant and only considered the source term W to vary randomly in time as a white noise process. Based on this assumptions, Liang (2013) derived the covariance, variance and spectrum of the groundwater heads with the first order perturbation equation using the linearized equation. In this work, we focus on the analytical solution of the head spectrum. For a detailed explanation of the derivation of the following equation we would like to refer to the paper from Liang (2013). The time independent head spectrum is given by: AGU - iPosterSessions.com

$$S_{hh}(x',\omega) = \frac{16}{\pi^2 S_y^2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n} B_m B_n S_{ww}}{(2m^2 + 2n^2 + 2m + 2n + 1)} \cdot \frac{(2m+1)^2}{(2m+1)^4 t_c^{-2} + \omega^2}$$

$$B_m = rac{cos[(2m+1)\pi x'/2]}{(2m+1)}, \qquad x' = rac{X}{L}$$

where t_c is the so called characteristic time of the aquifer (also referred to as characteristic time scale or aquifer response time), x is the position of the head measurement in the aquifer and L is the aquifer length from the water divide to the river. The analytical head spectrum for a white noise recharge is plotted in figure 1. A white noise process is temporally uncorrelated yielding a spectrum with fluctuations around a horizontal line for a single realization. As shown in the slide show, the aquifer serves as a low-pass filter which dampers higher frequencies and let lower frequencies pass with less alteration. Dependent on its properties, the aquifer filters the white noise leading to a break point in the spectrum which is related to the characteristic time of the aquifer. Increasing the storativity (figure 1 (a)), decreasing the transmissivity (figure 1 (b)) or extending the length of the aquifer (figure 1 (d)) results in higher characteristic times, meaning that perturbations remain a longer time in the aquifer until they have vanished, moves the break point towards lower frequencies and introduces an earlier scaling towards the higher frequencies. The position of the head measurement just influences the overall magnitude of the spectrum (figure 1 (c)).



Figure 1: Semi-analytical power spectra of a groundwater head fluctuation calculated with a Gaussian white noise recharge. In each subplot another parameter was varied. (a): storativity S (=specific yield S_{y}); (b): transmissivity T; (c): position along the transect x; (d): aquifer length. Except for varied parameters, remaining parameters are kept constant for each subplot: S = 0.01, T = 0.001 m²s⁻¹, x = 500 m, L = 1000 m. Vertical dashed lines mark frequencies which correspond to a period of 1 year, 1 month and 1 week, from left to right, respectively.

The characteristic time is defined by physical properties of the aquifer T, S and L, thus having a physical meaning. While many authors define t_c as the response behaviour of the aquifer, considering an aquifer recession constant (e.g. Gelhar 1974), it can also be related to and should not be confused with the time until an aquifer reaches the near-steady state (e.g. 95 % equilibrium) after a certain perturbation, t_{NE} (Rousseau-Gueutin 2013, Carr 2018). In this work we will refer to the characteristic time t_c for the Dupuit aquifer model to be consistent with the assumptions of the analytical solution from Liang (2013) using the following equation:

$$t_{Dup1} = \frac{4L^2 S_y}{\pi^2 T}$$

The characteristic time can vary over orders of magnitude and depends strongly on the aquifer length. It might range from a few weeks for small and permeable catchments with low specific storage to several years for large catchments with smaller transmissivities and higher storativities. For instance, if L = 1000 m, $S_y = 0.1$, T = 0.01 then $t_c = 47$ days and if L = 5000 m, $S_y = 0.01$, T = 0.001 then $t_c = 1,172$ days.



Motivation

- Projecting the response of regional aquifer systems to strongly changing conditions is paramount.

We need regional GW models!

- Regional groundwater models are difficult to parameterize!
- Usually aquifers are investigated through pumping tests.



Methodology

Spectral Analysis (SA)

Answer

Take GW level time series and derive the aquifer parameters from the spectral response!











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CONCLUSION

Conclusion

In this study we investigated the feasibility of the spectral analysis in four numerical settings with different complexity (i.e. aquifer properties) and forcing. Finally, we examined the sensitivity of this approach by using real groundwater data monitored in the Main catchment in central Germany. We conclude this work with following statements:

- 1. In synthetic, homogeneous environments the model input parameters can be precisely derived with spectral analysis of groundwater head fluctuations.
- 2. The mean vertical head of aquifers with little vertical flow components and gradients can be represented with a 1D Dupuit approximation.
- 3. Time series of groundwater head fluctuation must be roughly 10 times as long as the characteristic time t_c of the aquifer.
- 4. In stochastic ensembles of numerical models the effective transmissivity represents the geometric mean of the underlying distribution with a slight shift towards the harmonic mean. The variance of the derived parameters is much smaller when the heads are observed in the aquifer body or close to the water divide.
- 5. The derived aquifer parameters represent local as well as regional portions of the aquifer. Especially in settings where flow is forced to pass zones of lower hydraulic conductivity, regions with high hydraulic conductivity seams to disappear i.e. having less or even no effect on the hydraulic regime.
- 6. Precise parameter estimates can be obtained even for realistic, temporally correlated (generated by mHM model) recharge.
- 7. Application of the spectral analyses to real data grants reasonable results. Although complex flow regimes could be identified, the shape of the spectra follows the theoretical shape of a Dupuit aquifer for lower frequencies.
- 8. In a sensitivity analysis t_c turned out to be the most robust parameters even if L and x is changed over orders of magnitude. T and S compensate for this.
- 9. Applying the spectral analysis to groundwater head time series from a complex 3D numerical model, we were able to infer back the model input parameters with acceptable precision.

This analysis has shown that the results of the spectral analysis are reasonable but there is still need for validation. Future work will be tailored to validation as well as regionalization of the results of the spectral analysis.

Annotations

Most of the content of this iPoster has been extracted from a manuscript which is currently in process and will be submitted to AGU publication *Water Resources Research* soon.

The contributing authors are listed within each section of the iPoster.

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Research interests

- spectral analysis of baseflow and groundwater head fluctuations to derive effective regional aquifer parameters
- numerical groundwater modelling
- high performance computing
- · goal oriented modelling and complexity assessment
- machine learning

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ABSTRACT

Climate change is projected to have severe impacts on the amount and dynamics of groundwater recharge and consequently on water resources stored in aquifers. Projecting the response of regional aquifer systems to changing climate conditions using regional groundwater models is paramount. However, parameterizing the large-scale GW models are challenging due to scarcity of observational records as well as mismatch between scales of modeling and measurements.

In this work, we propose to bridge the scale gap and derive regional scale hydraulic parameters by spectral analysis of groundwater head fluctuations.

While the response of groundwater to external perturbations depends on local properties and local boundary conditions, it also contains signals of regional components due to the diffusive character of Darcy's law. We hypothesize that specific locations in aquifers can reveal regional parameters of the hydraulic system (e.g. aquifer transmissivity, storativity and characteristic time).

In order to proof our hypothesis, we first generate ensembles of synthetic but realistic aquifers which systematically differ in complexity with respect to their hydraulic heterogeneity. Applying Liang and Zhang's (2013) semi-analytical solution for the power spectrum of head data, we identify for each ensemble member and at different locations effective aquifer parameters. Next, we extend our study to investigate the use of spectral analysis in more complex numerical models and in real settings.

Our analyses indicate that the variance of inferred transmissivity and storativity values for the stochastic aquifer ensembles is small for observations points which are far away from the river boundary. Moreover, the head time series have to cover a period which is roughly ten times as long as the characteristic time of the aquifer. In deterministic aquifer models we infer effective regionally valid parameters which cannot be identified with classical pumping tests. Furthermore, the derived transmissivity and storativity values can be mapped to the distributed aquifer parameters of complex model domains. A sensitivity analysis further reveals that as long as aquifer length and position of the groundwater location is roughly known, transmissivity, storativity as well as the characteristic time can be robustly estimated.

REFERENCES

Carr, E. J. & Simpson, M. J. (2018): Accurate and efficient calculation of response times for groundwater flow, Journal of Hydrology, Elsevier BV, 2018, 558, 470-481

Gelhar, L. W. (1974): Stochastic analysis of phreatic aquifers, Water Resources Research, American Geophysical Union (AGU), 1974, 10, 539-545

Kolditz, O.; Bauer, S.; Bilke, L.; Böttcher, N.; Delfs, J. O.; Fischer, T.; Görke, U. J.; Kalbacher, T.; Kosakowski, G.; McDermott, C. I.; Park, C. H.; Radu, F.; Rink, K.; Shao, H.; Shao, H. B.; Sun, F.; Sun, Y. Y.; Singh, A. K.; Taron, J.; Walther, M.; Wang, W.; Watanabe, N.; Wu, Y.; Xie, M.; Xu, W. & Zehner, B. (2012): OpenGeoSys: an open-source initiative for numerical simulation of thermo-hydro-mechanical/chemical (THM/C) processes in porous media, Environmental Earth Sciences, Springer Nature, 2012, 67, 589-599

Liang, X. and Zhang, Y.-K. (2013): Analytic solutions to transient groundwater flow under time-dependent sources in a heterogeneous aquifer bounded by fluctuating river stage, Advances in Water Resources, vol. 58, pp. 1–9, Aug. 2013, doi: 10.1016/j.advwatres.2013.03.010.

Müller, S. (2020, April 2): GeoStat-Framework/ogs5py: v1.1.1 (Version v1.1.1). Zenodo. http://doi.org/10.5281/zenodo.3738563

Müller, S. & Schüler, L. (2020, April 14): GeoStat-Framework/GSTools: Volatile Violet v1.2.1 (Version v1.2.1). Zenodo. http://doi.org/10.5281/zenodo.3751743

Rousseau-Gueutin, P.; Love, A. J.; Vasseur, G.; Robinson, N. I.; Simmons, C. T. & de Marsily, G. (2013): Time to reach near-steady state in large aquifers, Water Resources Research, American Geophysical Union (AGU), 2013, 49, 6893-6908

Samaniego L., R. Kumar, S. Attinger (2010): Multiscale parameter regionalization of a grid-based hydrologic model at the mesoscale. Water Resour. Res., 46,W05523, doi:10.1029/2008WR007327,

Weiss, G. (1969): Statistical Theory of Communication. Y. W. Lee. Wiley, New York, Science, American Association for the Advancement of Science (AAAS), 1960, 132, 1546-1547

Yang, J.; Heidbüchel, I.; Musolff, A.; Reinstorf, F. & Fleckenstein, J. H. (2018): Exploring the Dynamics of Transit Times and Subsurface Mixing in a Small Agricultural Catchment, Water Resources Research, American Geophysical Union (AGU), 2018, 54, 2317-2335